Conditional probability

the probability that A occurs, given B

= P(A|B)

= P(A&B) / P(B) = the probability that A and B both occur, divided by the probability that B occurs

E.g. consider rolling a die.

P (you roll even | you roll 4) =

= the probability that you roll 4 *and* you roll an even number, divided by the probability that you roll 4 =

= (1/6) / (1/6) = 1

In other words, assuming that you rolled 4, it's 100% percent that you rolled even.

P (you roll 4 | you roll even) =

= the probability that you roll 4 *and* you roll an even number, divided by the probability that you an roll even number =

= (1/6) / (1/2) = 1/3

In other words, assuming that you rolled an even number, there's a 33.3% chance that you rolled 4. This makes sense, since you then rolled 2, 4, or 6.

Conditional probabilities tell us about the probabilities in a given (hypothetical) scenario. They filter out the probability of the scenario itself--we pretend that that the scenario is certain. As a result, the conditional probability of an event can be very different from its probability *simpliciter*. For example, suppose that the probability of a nuclear war in the near future is 1%. And suppose that the probability that a nuclear war breaks out once Trump has pushed the big red button is 99% (there's a tiny chance that the button malfunctions, or that the target fails to retaliate). Then we have that

P(nuclear war) = 0.01

P(nuclear war | Trump pushes the button) = 0.99

Conditionalizing can turn a low-probability event into a high-probability one, because we're considering its probability relative to an event which could make it very probable.

Using conditional probabilities in inference

We use experience to infer to the truth or falsity of hypotheses. If we have a hypothesis H (e.g. "God exists") and some relevant experience E (e.g. meeting the resurrected Jesus), then we have reason to believe H on the basis of E if

(1) P(hypothesis H | experience) > P(denial of H | experience)

Unfortunately, these conditional probabilities are not known to us, because we don't know how probable our experience is (we can't work that out without some prior idea about how the world is, which in turn, would require knowing whether H is true).

However, there are probabilities we can work out (more or less) and which can help us settle whether (1) is true. Specifically, we can sometimes guess the following probabilities:

P(experience | hypothesis H)

P(experience | denial of H)

For example, meeting the resurrected Jesus is, presumably, much more probable under the hypothesis that God exists than under atheism.

Using the definition of conditional probability, one can work out the following rules:

<u>Rule A</u>

(1) is true if the following are true:

(2) P(experience | hypothesis) > P(experience | denial of H)

<u>and</u>

(3) P(hypothesis H) > P(denial of H)

<u>Rule B</u>

(1) is true if the following are true:

(4) P(experience | hypothesis) >> P(experience | denial of H) (">>" means "much larger than")

<u>and</u>

(5) P(hypothesis H) is roughly equal to, or, at any rate, not significantly lower than, P(denial of H)

For example, the existence of God is more probable given that you met the resurrected Jesus if one of the following is true:

- (2) meeting the resurrected Jesus is more probable if God exists than if God doesn't exist <u>and</u> (3) the probability that God exists is higher than the probability that God doesn't exist
- (4) meeting the resurrected Jesus is *much* more probable if God exists than if atheism is true <u>and</u> (5) the probability that God exists is roughly equal to, or at least not significantly lower than, the probability that God doesn't exist

Proof: Let "D" be the denial of H.

We want to derive (1): P(H|E) > P(D|E).

P(H|E) > P(D|E) is true exactly if P(E&H) / P(E&D) > 1. Let's abbreviate this as "X > 1."

If P(E|H) > P(E|D) , then P(E&H) / P(E&D) > P(H)/P(D), in other words, X > P(H)/P(D).

So if P(H)/P(D) is greater than 1, i.e. if P(H) is higher than P(D), then (1) is true. And so we have Rule A.

Moreover, if P(E|H) >> P(E|D), then X >> P(H)/P(D). If, in addition, P(H)/P(D) is roughly 1 or, at least, not significantly lower than 1, then, again, X>1, and (1) is true. So we have Rule B.